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Weyl Invariant Gonihedric Strings

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Abstract

We study quantum corrections in the earlier proposed string theory, which is based on purely extrinsic curvature and Weyl invariant action. At the classical level the string tension in this theory is equal to zero and quarks viewed as open ends of the surface are propagating freely without interaction. We calculate one loop quantum corrections and demonstrate that quantum fluctuations generate nonzero area term (string tension).

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1 Introduction

In [1, 2] authors suggested so-called gonihedric model of random surfaces, which is based on the concept of extrinsic curvature. It differs in two essential points from the models considered in the previous studies [3, 4, 5, 6]: first, it claims that extrinsic curvature term alone should be considered as the fundamental action of the theory

$$S = m \cdot L = m \int d^2\zeta \sqrt{g} \sqrt{K^{ia}_a K^{ib}_b}, \tag{1}$$

here m has dimension of mass, K is a second fundamental form (extrinsic curvature) and there is *no area* term in the action. Secondly, it is required that the dependence on the extrinsic curvature should be such that the action will have dimension of length $L \propto \mathit{length}$, that is proportional to the linear size of the surface similar to the path integral [3]. When the surface degenerates into a single world line the functional integral over surfaces will naturally transform into the Feynman path integral for point-like relativistic particle

$$S = m\ L \quad \rightarrow \quad m \int_x^y ds. \tag{2}$$

At the classical level the string tension in this theory is equal to zero and quarks viewed as open ends of the surface are propagating freely without interaction $T_{classical} = 0$ because the action (1) is equal to the perimeter of the flat Wilson loop $S \rightarrow m(R+T)$. It was demonstrated in [1] that quantum fluctuations generate the area term A in the effective action

$$S_{eff} = m\ L \quad + \quad T_q\ A \quad + \quad ..., \tag{3}$$

with the non-zero string tension $T_q = \frac{D}{a^2}\ (1 - ln\frac{d}{\beta})$, here D is the dimension of the spacetime, a is a scaling parameter. In the scaling limit $\beta \rightarrow \beta_c = D/e$ the string tension has a finite limit. Therefore at the tree level the theory describes free quarks with string tension equal to zero, instead quantum fluctuations generate nonzero string tension and, as a result, quark confinement. The theory may consistently describe asymptotic freedom and confinement as it is expected to be the case in QCD.

The quantum effect of string tension generation (3) has been found in the discrete formulation of the theory when the action (1) is written for the triangulated surfaces. Our aim now is to show that this theory is well defined in one loop approximation and that similar generation of non-zero string tension takes place in a continuum formulation of the theory (1) and therefore may lead to a nontrivial string theory. Here we shall treat quantum fluctuations in two different ways following the works of Polyakov [3] and Kleinert [4].

2 Weyl Invariance

We shall represent the gonihedric action (1) in a Weyl invariant form

$$S = m \int d^2\zeta \sqrt{g} \sqrt{(\Delta(g) X_\mu)^2}, \tag{4}$$

[3] This is in contrast with the previous proposals when the extrinsic curvature term is a scale invariant $\vec{X} \rightarrow \lambda \vec{X}$ functional $F(\mathit{extrinsic\ curvature}) \propto 1$.

here $g_{ab} = \partial_a X_\mu \partial_b X_\mu$ is induced metric, $\Delta(g) = 1/\sqrt{g}\ \partial_a \sqrt{g} g^{ab} \partial_b$ is a Laplace operator and $K_a^{ia} K_b^{ib} = (\Delta(g) X_\mu)^2$. The second fundamental form K is defined through the relations:

$$
\begin{aligned}
K^i_{ab} n^i_\mu &= \partial_a \partial_b X_\mu - \Gamma^c_{ab} \partial_c X_\mu = \nabla_a \partial_b X_\mu, & (5)\\
n^i_\mu n^j_\mu &= \delta_{ij}, \quad n^i_\mu \partial_a X_\mu = 0, & (6)
\end{aligned}
$$

where n^i_μ are $D-2$ normals and $a, b = 1, 2;\qquad \mu = 0, 1, 2, ..., D-1;\quad i, j = 1, 2, ..., D-2.$

We can introduce independent metric coordinates g_{ab} using standard Lagrange multipliers λ^{ab} and then fix conformal gauge $g_{ab} = \rho \delta_{ab}$ using reparametrization invariance. After fixing conformal gauge we shall have

$$
S = m \int d^2\zeta \left\{ \sqrt{(\partial^2 X^\mu)^2} + \lambda^{ab} \left(\partial_a X^\mu \partial_b X^\mu - \rho \delta_{ab} \right) \right\}. \tag{7}
$$

As one can see the first term is ρ independent and therefore is explicitly Weyl invariant. Note that extrinsic curvature part of the Polyakov-Kleinert action $S_{PK} = \frac{1}{e} \int d^2\zeta \sqrt{g} K_a^{ia} K_b^{ib}$ is not Weyl invariant. This action leads to the following equation of motion:

$$
\partial^2 \frac{\partial^2 X_\mu}{\sqrt{(\partial^2 X_\nu)^2}} - \partial_a \left(\lambda^{ab} \partial_b X_\mu \right) = 0, \quad \partial_a X_\mu \partial_b X_\mu - \rho \delta_{ab} = 0, \quad \lambda^{aa} = 0. \tag{8}
$$

In the light cone coordinates $\zeta^\pm = (\zeta^0 \pm \zeta^1)/\sqrt{2}$ the conformal gauge looks like:

$$
g_{++} = g_{--} = 0, \qquad g_{+-} = \rho, \tag{9}
$$

the connection has only two nonzero components $\Gamma^+_{++} = \partial_+ \ln \rho,\ \Gamma^-_{--} = \partial_- \ln \rho$ and metric variation is:

$$
\delta g_{\pm\pm} = \nabla_\pm \varepsilon_\pm = \rho \partial_\pm \varepsilon^\mp. \tag{10}
$$

In these coordinates the partition function takes the following form:

$$
Z = \int \exp \imath \{S\} \det\{\nabla_+\} \det\{\nabla_-\} DX^\mu D\lambda_{ab} D\rho, \tag{11}
$$

where $\det\{\nabla_+\} \det\{\nabla_-\}$ are Faddeev-Popov determinants corresponding to the conformal gauge (9).

To obtain quantum correction to the classical action we have to expand our action around some classical solution $(\bar{X}^\mu, \bar{\lambda}^{ab}, \bar{\rho})$ up to second order on small fluctuations $(X_1^\mu, \lambda_1^{ab}, \rho_1)$, thus $S = \bar{S} + S_2 + S_{int}$, where $\bar{S} = m \int d^2\zeta \sqrt{\bar{n}^2}$,

$$
S_2 = \int \frac{m}{2\sqrt{\bar{n}^2}} X_1^\mu \partial^4 \left(\eta^{\mu\nu} - \frac{\bar{n}_\mu \bar{n}_\nu}{\sqrt{\bar{n}^2}} \right) X_1^\nu d^2\zeta + m \int (2\lambda_1^{ab}\ \bar{e}^a_\mu\ \partial_b X_1^\mu - \lambda_1^{aa} \rho_1) d^2\zeta,
$$

$$
S_{int} = m \int d^2\xi (\bar{\lambda}^{ab} \partial_a X_1^\mu \partial_b X_1^\mu + \lambda_1^{ab} \partial_a X_1^\mu \partial_b X_1^\mu + ...), \tag{12}
$$

where we have introduced convenient notations

$$
\bar{n}_\mu = \partial^2 \bar{X}_\mu, \qquad \bar{e}^a_\mu = \partial_a \bar{X}_\mu.
$$

Here we admit slow behavior of the first and second derivatives of the classical solution $\bar{X}^\mu$ and natural separation of interaction with external field $\bar{\lambda}^{ab}$.

One can see from (12) that in one loop approximation we have factorization of tangential modes of X_1^μ and cancellation of their contribution with the corresponding ghost determinant. To see that, we shall expand X_1^μ into tangential and normal fields $\phi_a,\ \xi^i$:

$$\begin{aligned} X_{1\mu} &= \phi_a\ \bar{e}^a_\mu + \xi^i\ \bar{n}^i_\mu, &(13)\\ \bar{n}^i_\mu\ \bar{e}^a_\mu &= 0, \quad \bar{e}^a_\mu\ \bar{n}_\mu = 0. &(14)\end{aligned}$$

The last relation, together with the following ones $\bar{n}_\mu \bar{n}^i_\mu = \bar{K}^{ia}_a \equiv \bar{K}^i,\ \bar{n}_\mu \bar{n}_\mu = \bar{K}^{ia}_a\ \bar{K}^{ib}_b \equiv \bar{K}^2$, can be easily seen in conformal gauge (9). We also have

$$\bar{S} = m \int d^2\zeta \sqrt{\bar{K}^2}. \tag{15}$$

We have to substitute expansion (13) into (12) and take into account slow behavior of classical fields compared with quantum fields:

$$\begin{aligned} S_2 = \int d^2\zeta \Big\{ &\frac{m}{2\sqrt{\bar{K}^2}}\ \xi^i\ \partial^4 \left(\delta^{ij} - \frac{\bar{K}^i\bar{K}^j}{\bar{K}^2}\right) \xi^j \\ &+ \lambda_1^{\pm\pm} \nabla_\pm \phi_\pm \ +\ \lambda_1^{+-}\rho_0 \left(\partial_+\phi^+ + \partial_-\phi^-\right) - \lambda_1^{+-}\rho_1 \Big\}, \\ S_{int} = m \int d^2\zeta &\bar{\lambda}^{ab} \left(\partial_a\xi^i\partial_b\xi^i + \partial_a\phi^c\partial_b\phi_c\right). \end{aligned} \tag{16}$$

Using the last expression for S_2 in partition function (11) and integrating it over ρ_1 and $\lambda_1^{\pm\pm}$ we shall get delta functions $\delta(\lambda_1^{+-})\delta(\nabla_\pm\phi_\pm)$. Then integrating over λ_1^{+-} and longitudinal components $\phi^\pm$ we shall get determinants $det^{-1}\nabla_\pm$. We observe now that in the one loop approximation there is a cancellation of these determinants with the ghost determinants in (11) and therefore absence of conformal anomalies. This should be verified in the next order where we have to consider third order interactions of the form $\lambda_1^{ab}\partial_a X_1^\mu \partial_a X_1^\mu$.

Finally we have the following one loop partition function

$$Z_1 = exp(\ i\ \bar{S}\ ) \int exp\Big\{\ i\ S_2(\bar{K},\xi^i) + i\ S_{int}(\bar{\lambda},\xi^i)\ \Big\} D\xi^i, \tag{17}$$

where

$$\begin{aligned} S_2 &= \int d^2\zeta\ \frac{m}{2\sqrt{\bar{K}^2}}\ \xi^i\ \partial^4 \left(\delta^{ij} - \frac{\bar{K}^i\bar{K}^j}{\bar{K}^2}\right) \xi^j, &(18)\\ S_{int} &= m \int d^2\zeta\ \bar{\lambda}^{ab}\ \partial_a\xi^i\partial_b\xi^i. &(19)\end{aligned}$$

From quadratic part (18) we can deduce the propagator

$$\langle \xi^i(p)\xi^j(-p)\rangle = \frac{\sqrt{\bar{K}^2}}{m}\frac{\Pi^{ij}}{(p^2)^2}, \qquad \Pi^{ij} = \delta^{ij} - \frac{\bar{K}^i\bar{K}^j}{\bar{K}^2}, \quad \Pi^{ii} = D-3 \tag{20}$$

and calculate first correction to the classical action (15). For that we have to contract ξ^i fields in (19) using (20)

$$W_1 = m \int d^2\zeta\ \bar{\lambda}^{ab}\ \langle \partial_a\xi^i\partial_b\xi^i\rangle = \frac{D-3}{2\pi} \log\left(\Lambda/\tilde{\Lambda}\right) \int d^2\zeta\ \bar{\lambda}^{aa}\sqrt{\bar{K}^2}, \tag{21}$$

thus $m_1 = m + \bar{\lambda}^{aa} \frac{D-3}{2\pi} \log\left(\Lambda/\tilde{\Lambda}\right)$. On the classical trajectory (8) we have $\bar{\lambda}^{aa} = 0$ and therefore there is no quantum corrections on-mass shell S-matrix elements and the theory is finite. The absence of quantum corrections at one loop level is very similar to the pure quantum gravity in four dimensions [7]. One can expect now that summation of all one loop diagrams with external field $\bar{\lambda}^{ab}$ may generate nontrivial solution and condensation of Lagrange multiplier $\bar{\lambda}^{aa} \neq 0$ on quantum level [8, 9, 10, 11].

3 One loop Effective Action

Our aim here is to sum up all one loop diagrams in the $\bar{\lambda}^{ab}$ background. For that we have to keep all one loop diagrams which are induced by the vertex $\bar{\lambda}^{ab}\ \partial_a \xi^i \partial_b \xi^i$. Integrating (17) together with the interaction term one can get [9, 10]

$$W_1 = \frac{i}{2}\ Trln\bar{H}, \tag{22}$$

where

$$\tilde{H}_{ij} = \Pi_{ij}\ \left\{\ (\partial^2)^2 - 2\sqrt{\bar{K}^2}\ \bar{\lambda}^{ab}\partial_a\partial_b\ \right\}. \tag{23}$$

We are looking for the solution in the form

$$\bar{\lambda}^{ab} = \lambda\ \delta^{ab}, \tag{24}$$

where λ is a constant field. Then

$$\tilde{H}_{ij} = \Pi_{ij}\ \left\{\ (\partial^2)^2 - 2\sqrt{\bar{K}^2}\ \lambda\ \partial^2\ \right\}. \tag{25}$$

It is convenient to introduce the notation $P_\alpha = i\partial_\alpha$ and we shall factor this operator into two pieces $\tilde{H} = H * H_0$, where

$$H_{ij} = -\Pi_{ij}\left(P^2 + 2\lambda\ \sqrt{\bar{K}^2}\right), \qquad H_{0ij} = -\Pi_{ij}P^2. \tag{26}$$

The effective action takes the form

$$L_1 = -\frac{i}{2}\int\frac{ds}{s}TrU(s)\ -\ \frac{i}{2}\int\frac{ds}{s}TrU_0(s), \tag{27}$$

where trace Tr is over Lorentz and world sheet coordinates ζ and

$$U(s) = exp(-iHs), \qquad U_0(s) = exp(-iH_0 s). \tag{28}$$

Taking the trace over Lorentz indexes and using the matrix element $(\zeta'(s)|\zeta''(0)) = (\zeta'|e^{iP^2 s}|\zeta'') = \frac{1}{4\pi s}exp\left(-i\frac{(\zeta''-\zeta')^2}{4s}\right)$ one can get

$$L_1 = -\frac{i}{2}\int\frac{ds}{s}\left(\frac{D-3}{4\pi s}e^{2i\lambda s\sqrt{\bar{K}^2}} + \frac{D-3}{4\pi s}\right) \tag{29}$$

and after rotation of the contour by $s \to -is$ and substraction of vacuum contribution we shall get

$$L_1 = \frac{D-3}{8\pi}\int\frac{ds}{s^2}\left(e^{-2\lambda s\sqrt{\bar{K}^2}} - 1\right) + C\lambda. \tag{30}$$

The counterterm is equal to $C\lambda$ and we can fix it by normalization condition [8, 9]:

$$\frac{\partial L_1}{\partial \lambda}|_{\lambda=m} = \sqrt{\bar{K}^2}, \tag{31}$$

after which we can get ultraviolet and infrared finite effective action in the form

$$L_1 = \tilde{\lambda}/2 + \frac{D-3}{8\pi} \int_0^\infty \frac{ds}{s^2} \left(e^{-\tilde{\lambda}s} - 1 + \tilde{\lambda} s e^{-\tilde{m}s} \right), \tag{32}$$

where $\tilde{\lambda} = 2\ \lambda\ \sqrt{\bar{K}^2}$ and $\tilde{m} = 2\ m\ \sqrt{\bar{K}^2}$. This expression can be integrated out and we shall get

$$L_1 = \tilde{\lambda}/2\ +\ \frac{D-3}{8\pi}\ \tilde{\lambda}\ [\ ln(\frac{\tilde{\lambda}}{\tilde{m}}) - 1\ ] \quad = \quad \lambda\ \sqrt{\bar{K}^2}\ +\ \frac{D-3}{4\pi}\ \lambda\ \sqrt{\bar{K}^2}\ [\ ln(\frac{\lambda}{m}) - 1\ ] \tag{33}$$

with its new minimum at the point

$$< \lambda >= m\ exp(-\frac{4\pi}{D-3}). \tag{34}$$

4 Consideration in the Physical Gauge

To study quantum effects from a different perspective we shall consider only normal, physical perturbation of the world sheet $X_\mu(\zeta)$ [4, 12]

$$\tilde{X}_\mu = X_\mu + \xi^i n^i_\mu, \tag{35}$$

then the first derivative is $\partial_a \tilde{X}_\mu = \partial_a X_\mu + \partial_a \xi^i n^i_\mu + \xi^i \partial_a n^i_\mu$ and the metric is equal to

$$\tilde{g}_{ab} = g_{ab} + \xi^i \partial_a X_\mu \partial_b n^i_\mu + \xi^i \partial_b X_\mu \partial_a n^i_\mu + \partial_a \xi^i \xi^j n^i_\mu \partial_b n^j_\mu \tag{36}$$

$$+\xi^i \partial_b \xi^j \partial_a n^i_\mu n^j_\mu + \partial_a \xi^i \partial_b \xi^i + \xi^i \xi^j \partial_a n^i_\mu \partial_b n^i_\mu, \tag{37}$$

then it follows that

$$\tilde{g}^{ab} = g^{ab} + 2K^{iab}\xi^i - \nabla^a \xi^i \nabla^b \xi^i + 3K^{ia}_c K^{jcb} \tag{38}$$

and that

$$\tilde{g}^{1/2} = g^{1/2}[1 - \xi^i K^{ia}_a + \frac{1}{2} R^{ij} \xi^i \xi^j + \frac{1}{2} g^{ab} \nabla_a \xi^i \nabla_b \xi^j], \tag{39}$$

where

$$K^i_{ab} = \partial_a \partial_b X_\mu\ n^i_\mu, \qquad R^{ij} = K^{ia}_a K^{jb}_b - K^{ia}_b K^{jb}_a. \tag{40}$$

For the second derivative we have $\partial_a \partial_b \tilde{X}_\mu = \partial_a \partial_b X_\mu + \partial_a \partial_b \xi^i n^i_\mu + \partial_b \xi^i \partial_a n^i_\mu + \partial_a \xi^i \partial_b n^i_\mu + \xi^i \partial_a \partial_b n^i_\mu$. In order to compute the variation of the extrinsic curvature we have to find the perturbation of the normals $\delta n^i_\mu = \tilde{n}^i_\mu - n^i_\mu$, where $\tilde{n}^i_\mu \partial_a \tilde{X}_\mu = 0$, $\tilde{n}^i_\mu \tilde{n}^j_\mu = \delta^{ij}$. From this it follows that

$$\partial_a \xi^i + \delta n^i_\mu \partial_a X_\mu + \delta n^i_\mu (\partial_a n^j_\mu \xi^j + n^j_\mu \partial_a \xi^j) = 0, \qquad n^i_\mu \delta n^j_\mu + n^j_\mu \delta n^i_\mu + \delta n^i_\mu \delta n^j_\mu = 0 \tag{41}$$

and we can always represent δn^i_μ in the form $\delta n^i_\mu = A^{ia}\partial_a X_\mu + B^{ij} n^j_\mu$. The solution of (41) is

$$\delta n^i_\mu = -\partial^a X_\mu (\partial_a \xi^i + K^{jb}_a \xi^j \partial_b \xi^i) - \frac{1}{2} n^j_\mu \partial^a \xi^j \partial_a \xi^i. \tag{42}$$

With this formulas we can get the variation of the extrinsic curvature up to the second order

$$\tilde{K}^i_{ab} = \left(\partial_a\partial_b X_\mu + \partial_a\partial_b \xi^i n^i_\mu + \partial_b \xi^i \partial_a n^i_\mu + \partial_a \xi^i \partial_b n^i_\mu + \xi^i \partial_a \partial_b n^i_\mu\right)(n^i_\mu + \delta n^i_\mu), \tag{43}$$

substituting (42) one can get for the variation $\delta K^i_{ab} = \tilde{K}^i_{ab} - K^i_{ab}$

$$\delta K^i_{ab} = \nabla_a \nabla_b \xi^i - K^{ic}_a K^j_{cb} \xi^j + \nabla_b K^{jc}_a \; \xi^j \nabla_c \xi^i \tag{44}$$

$$+(K^j_{bc} \nabla_a \xi^j + K^j_{ac} \nabla_b \xi^j) \nabla^c \xi^i - \frac{1}{2} K^j_{ab} \nabla^c \xi^j \nabla_c \xi^i. \tag{45}$$

Using (38) we can get the trace of the extrinsic curvature

$$\delta K^{ia}_a = \nabla^a \nabla_a \xi^i + K^{ic}_a K^{ja}_c \xi^j + \nabla^a K^j_{ab} \; \xi^j \nabla^b \xi^i + 2K^j_{ab} \nabla^a \xi^j \nabla^b \xi^i - K^i_{ab} \; \nabla^a \xi^j \nabla^b \xi^j \tag{46}$$

$$+K^j_{ab} K^{ib}_c K^{nca} \xi^j \xi^n - \frac{1}{2} K^{ja}_a \nabla^c \xi^j \nabla_c \xi^i + 2K^j_{ab} \xi^j \nabla^a \nabla^b \xi^i. \tag{47}$$

Using the last expression it is easy to compute the first and the second variations of the action (1):

$$\delta_1 A_{Gonihedric} = m \int d^2\zeta \sqrt{g}\; \xi^i \{\delta^{ij} \triangle + K^{ib}_a K^{ja}_b - \delta^{ij}\; K^2\} \frac{K^{ja}_a}{\sqrt{K^2}}, \tag{48}$$

$$\delta_2 A_{Gonihedric} = \frac{m}{2} \int d^2\zeta \sqrt{g} \frac{1}{\sqrt{K^2}} \{\triangle \xi^i \triangle \xi^i - \frac{K^i K^j}{K^2} \triangle \xi^i \triangle \xi^i \tag{49}$$

$$+K^i K^j \nabla^c \xi^i \nabla_c \xi^j + K^i K^i \nabla^c \xi^j \nabla_c \xi^j - 2K^{ib}_a K^{ja}_b \nabla^c \xi^i \nabla_c \xi^j \tag{50}$$

$$-2K^i K^i_{bc} \nabla^b \xi^j \nabla^c \xi^j + \frac{2}{K^2} K^n K^{nb}_a K^{ja}_b K^i \nabla^e \xi^i \nabla_e \xi^j\}, \tag{51}$$

where $K^i = K^{ia}_a$ and $K^2 = K^{ia}_a K^{ib}_b = K^i K^i$. The equation of motion is

$$\{\delta^{ij} \triangle - R^{ij}\} \frac{K^j}{\sqrt{K^2}} = 0, \tag{52}$$

where $R^{ij} = K^i K^j - K^{ib}_a K^{ja}_b$. It is easy to see that

$$< \nabla_a \xi^i \nabla_b \xi^j >= \delta_{ab}\; \Pi^{ij}\; \frac{\sqrt{K^2}}{m}\; \frac{1}{2\pi}\; \log\left(\Lambda/\tilde{\Lambda}\right), \tag{53}$$

where $\Pi^{ij} = \delta^{ij} - \frac{K^i K^j}{K^2}$. Using (53) one can find the one loop contribution

$$\frac{m}{2\sqrt{K^2}}\; K^i K^j\; 2\; \Pi^{ij}\; \frac{\sqrt{K^2}}{m} \frac{1}{2\pi}\; \log\left(\Lambda/\tilde{\Lambda}\right) \tag{54}$$

$$+\frac{m}{2\sqrt{K^2}}\; K^2\; 2\; (D-3)\; \frac{\sqrt{K^2}}{m} \frac{1}{2\pi}\; \log\left(\Lambda/\tilde{\Lambda}\right) \tag{55}$$

$$+\frac{m}{2\sqrt{K^2}}\ (-2)\ K^{ib}_aK^{ja}_b\ 2\ \Pi^{ij}\ \frac{\sqrt{K^2}}{m}\frac{1}{2\pi}\ \log\left(\Lambda/\tilde{\Lambda}\right) \tag{56}$$

$$+\frac{m}{2\sqrt{K^2}}\ (-2)\ K^2\ (D-3)\ \frac{\sqrt{K^2}}{m}\ \frac{1}{2\pi}\ \log\left(\Lambda/\tilde{\Lambda}\right) \tag{57}$$

$$+\frac{m}{2\sqrt{K^2}}\ \frac{2}{K^2}\ K^nK^{nb}_aK^{ja}_bK^i\ 2\ \Pi^{ij}\ \frac{\sqrt{K^2}}{m}\frac{1}{2\pi}\ \log\left(\Lambda/\tilde{\Lambda}\right) \tag{58}$$

The first and the last terms are equal to zero because $\Pi^{ij}K^j = 0$, the second and the fourth terms cancel each other and we see that *in this theory the counterterm does not depend on the dimension of the embedding space time*. Only the third therm is non zero

$$-2\ K^{ib}_aK^{ja}_b\ \Pi^{ij}\ \frac{1}{2\pi}\ \log\left(\Lambda/\tilde{\Lambda}\right) = 2(K^iK^j - R^{ij})\ \Pi^{ij}\ \frac{1}{2\pi}\ \log\left(\Lambda/\tilde{\Lambda}\right) = -2R\ \frac{1}{2\pi}\ \log\left(\Lambda/\tilde{\Lambda}\right), \tag{59}$$

here we have used equation of motion $R^{ij}K^j = 0$. Thus it is proportional to the Euler characteristic of the surface and can be neglected. This is completely consistent with our previous result that the theory is finite at one loop level.

It is instructive to compare this result with the quantum corrections in scale invariant theory [3, 4]. The second variation of the Polykov-Kleinert action is

$$\delta A_{PK} = \frac{1}{2e_0}\int d^2u\sqrt{g}\{\triangle\xi^i\triangle\xi^i + K^{ia}_aK^{jb}_b\nabla^c\xi^i\nabla_c\xi^j + \frac{1}{2}K^{ia}_aK^{ib}_b\nabla^c\xi^j\nabla_c\xi^j \tag{60}$$

$$-2K^{ib}_aK^{ja}_b\nabla^c\xi^i\nabla_c\xi^j - 2K^{ia}_aK^{i}_{bc}\nabla^b\xi^j\nabla^c\xi^j\}, \tag{61}$$

and the one-loop correction renormalizes the coupling constant

$$\frac{1}{e} = \frac{1}{e_0} - \frac{D}{4\pi}\log\left(\Lambda/\tilde{\Lambda}\right). \tag{62}$$

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